# Application of the Superposition Method to the Coreless Induction Furnaces

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**Abstract**— the coreless induction furnace is one of the most important applications of induction heating, because of its wide utility in large industrial plants. This paper will discuss the electrical characteristics of coreless induction furnace by the superposition method. Also, a comparison was done between practical and theoretical ampere-turn arising from the superposition method. The percentage of error between theoretical and practical case will reach the minimum value 5.47%, (when the input power is 0.183 kW), as being closet to the optimum frequency of operation under certain circumstances of current, number of turns, load... etc.

**Key words**— Induction Heating, coreless furnace, Superposition method.

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## 1 INTRODUCTION

The coreless furnace [1], as shown in Fig. (1) is one of the most important application of the induction heating theory, which employed the same principle of electrical power transformer. The primary winding coil surrounds a refractory crucible containing pieces of metal charge, which is the secondary winding. The majority of magnetic flux produced by the primary winding flows in the charge, the resultant induced voltage and current (eddy current) causing it to heat [2].

To solve the electromagnetic problems of induction heating, there are two types of solution. First, analytical solutions, (Equivalent circuit method, approximate coil design method, Scale-model analogue method [2] and Superposition method [3, 4]). Second, numerical solutions (Finite difference method (FDM), Finite element method (FEM), mutually coupled circuit method, Boundary element method (BEM) and Hybrid finite element-boundary element method) [5, 6].

Among the important limitations to choose the proposed method is the fast running control time of furnace system and the suitability to all load – applications (for magnetic and non-magnetic) which mean inexpensive microprocessor and small storage unit memory, which is main point for industry. This was accomplished by the superposition method [4]. So, computer program was designed to represent this method and its development, and a comparison was made between practical and theoretical results.

## 2 EQUIVALENT CIRCUIT METHOD

The basic assembly of an induction heater consists of a water cooled copper coil surrounding a work piece. The coil can be designed by obtaining the values of the resistances and reactance, and solving the equivalent circuit. This method was devised by Baker [7].

The magnetic flux produced by the work coil, T, has alternative parallel paths through either, the work piece w, the coil c and the air gap g, as shown in Fig. (2).This magnetic situation is converted to the electrical equivalent circuit of Fig. (3), which can be solved by circuit analysis to give work coil turns, the current carrying capacity, efficiency and power factor. It should be mentioned here that the components of the equivalent circuit contain empirical factors with unspecified values.

## 3 SUPERPOSITION METHOD

This method depends on the calculation of the surface magnetic field distribution produced by a single current carrying conductor, as described briefly in [3]. Then, the superposition principle is applied to determine the magnetic field and power density produced by a number of conductors.

Although the superposition method was applied to induction heating, it has never been applied to induction melting. In this paper, this method will be applied to coreless induction furnace.

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It was concluded after several experiments being done, that there is somewhat little deference between the practical readings and that of the theoretical, under certain circumstances, which comprises the need to a dope the superposition general equation by some factors in order to be more practical and to get more accuracy in its calculations.

Thus the general equation derived to a conductor near a semi- infinite slab as shown in fig.(4) should be amended to the practical equation, so for a number of conductors (n), the magnetic field intensity at any point (p) is:

$$Hp = \frac{Ih_e}{\pi}\sum_{x=1}^{N-1}\frac{\beta}{{h_e}^2 - \frac{\alpha}{\cos\theta}(z_1+xs)^2} \qquad (1)$$

Where α and β are functions of the effective height he, α is a linear function of he, and it was found to be a straight line expressed by the following equation:

$$\alpha = 10^{-2}he \qquad (2)$$

And β value was found experimentally to be of the following equation:

$$\beta = -(2.642 * 10^{-1}) + (7.736 * 10^{-2}) - (1.539 * 10^{-3}){h_e}^2 + (1.31 * 10^{-5}){h_e}^3 - (3.968 * 10^{-8}){h_e}^4 \qquad (3)$$

While cos θ can be calculated from the following equation:

$$\cos\theta = \frac{h}{\sqrt{h^2+z^2}} \qquad (4)$$

Where θ : is the angle between the line, joining the conductor and point P, and the perpendicular line from the conductor to the slab as shown in fig.(4).

The effective height can be calculated from the equation below:

$$h_e = \sqrt{h^2 - r^2} \qquad (5)$$

According to references [2] and [3], it is valid to treat a circular conductor around a cylinder as a line current, at a distance from a flat plane, and the proceeding equations may be applied to a single turn coil, surrounding a cylinder, provided that the distance between the conductor and the cylinder , h , is :-

1- Small in comparison with the charge material radius to be melted. And,

2- Large in comparison with the current skin depth in the charge.

In order to have good imagination about the proposed method used to analyze the coreless induction furnace, it is necessary to make a sufficient comparison between the theoretical values of ampere-turn obtained from superposition method with practical values of ampere-turn obtained experimentally from reference [8] with a prototype coreless induction furnace, table (1). This was regarded as an indicator of how much this method can be applied efficiently on induction furnace, and also to give the percentage error between the two cases, which helps very much in obtaining the "best frequency" for this case under the specified circumstances i.e. number of turn, current, load… etc. This comparison is shown in table (2), which indicates a good correlation with the increase of the applied frequency. This result agrees with the derivation of the superposition method (small skin depth) i.e. high frequency.

The method studied the electrical characteristics of the coreless induction furnace [9] by giving each electrical parameter (i.e. Magnetic field (H), surface current density (J), and power density (PD)) along the furnace length using Superposition method [3]. The following calculations were carried out:-

1-Magnetic field intensity at any point Hp (A/m), is calculated from the equations (1):

(A) Without α, β and Cos θ yield:

$$\mathrm{Hp} = \frac{\mathrm{Ih_e}}{\pi}\sum_{x=0}^{N-1}\frac{1}{{\mathrm{h_e}}^2 + (\mathrm{z_1}+\mathrm{xs})^2} \qquad (6)$$

(B) With α, β and Cos θ as in equation (1).

3-Surface current density at any point Jp (A/m2) is calculated from:

$$\mathrm{J_p} = \frac{\sqrt{2}}{\delta}\mathrm{H_p} \qquad (7)$$

4-Power density at any point PD(w/m2) is calculated from:

$$\mathrm{P_D} = \frac{1}{2}\rho\delta{\mathrm{J_p}}^2 \qquad (8)$$

## 4 RESULTS AND DISCUSSION

This paper will discuss the electrical characteristics of coreless induction furnace by a program called "Compute-Result". All calculations were made to coreless induction furnace with dimensions as stated in table (1) [8]. More details for result discussions can be found in reference [9].

The computer results are shown in Figures. (5) to (12) for different, frequencies, currents and number of turns, all curves were drawn by Excel.

The magnetic field and power density distributions, along the furnace length, are shown in Figs (5) and (6).

The relation between the magnetic field strength (H) and the power density (PD) respectively versus the current (I) shown in Figs. (7) and (8). The relation between the magnetic field strength (H) and the power density (PD) respectively versus the number of turn (N) is shown in Figs. (9) and (10). The relation between the magnetic field strength (H) and the power density (PD) respectively versus the frequencies (f) showed in figures (11) and (12).

The following results were concluded from the proposed cases. As current doubles - with constant number of turns , the magnetic field strength along charge will also doubles, while the power density will increase by four times its value, see figures(7) and (8).

As number of turns doubles – with constant current– ,the magnitude of magnetic field strength along charge is increased by almost 1.8 times, while the power density is increased by 3.8 times, see figures(9) and (10).

When doubling both current and number of turns, the magnetic field along charge will increase by almost 3.7 times, while the power density is increased by almost 15.2 times its initial value, see figures(11) and (12).

The superposition method has high accuracy in its calculations, because it does not assume uniform magnetic field along the charge, which make it effectively applied to applications which require non-uniform densities in the load. Also the superposition method doesn't assume "uniform material properties", which is not correct, because the resistively of the charge changes with the temperature, also the permeability varies with both magnetic field strength and temperature.

In real case, the magnitude of magnetic field strength (H) along charge is less than the theoretical case by almost 2.5 times its value, while the power density (PD) along charge, at real case in induction furnace, is always less than the theoretical case by almost 6 times its value, because of the unexpected limiting surrounding circumstances such as environment temperature, current harmonics.

It can be deduced that power density is directly proportional with frequency. When the frequency is high, the power density will concentrate at the surface area part of the charge, i.e. applying high power on this specified area which gives faster melting time rate to this part. Then the later parts will be melted in turn by conduction and convection the heating energy to all other deepest parts of charge.

Then a comparison was done by a second program called "Ampere-Turn" which compute the theoretical Ampere-Turn arising from the Superposition method, so as to be compared with practical Ampere-Turn taken from reference [8], by table (2), three cases can be deduced, that for all three cases there are difference between practical and theoretical, and the practical is always less than the theoretical. Because the determination of the number of turns of the coil is exclusively influenced by practical considerations, since the designed power supply of the reference practical work doesn't have the ability to produce high currents, so it was compensated by using a coil of large number of turns of small diameter tube.

Also it was concluded from the difference in the percentage – error of the above three cases, that this value will decrease gradually with increasing the frequency until reaching the minimum value (in this case 5.47%) , as being closet to the optimum value of operating frequency under certain circumstances of current, load , number of turns… etc. (in this case 13000 Hz).

## 5 CONCLUSIONS

The induction furnace is one of the most important applications of the induction heating, in which both magnetic and non-magnetic metals can be melted.

There is no effect of frequency on the magnetic field strength characteristics along the charge, because the field strength equation (1) do not be affected by frequency changing at all, see Figure (11).

The main advantage of the superposition method is its simplicity, which permits build noncomplex programs with rapid running control time for the furnace system, i.e. inexpensive microprocessor and small storage unit memory, which is main point for industry. The superposition has high accuracy in its calculations.

The percentage of error between theoretical and practical case will reach the minimum value (in this case 5.47%), as being closet to the optimum frequency of operation under certain circumstances of current, number of turns, load… etc. (in this case 13000Hz).

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## Symbols

Hp : Magnetic field intensity (A/m).
I : Current (A).
$h_e$ : Effective height of conductor from the slab (m).
h : distance conductor center to the slab surface (m).
N : Number of turns.
z : distance along the surface between points P and 0, see fig. (4).
r : Circular conductor radius (m).
δc : Skin depth (m).
ρ : resistivity (ohm/m).

**TABLE 1**

Dimensions of Coreless Induction Furnace [8].

| | |
|---|---|
| Charge diameter | 37.31 mm |
| Charge height | 65 mm |
| Coil inner diameter | 51.01 mm |
| Coil height | 65 mm |
| No. of turn | 16 |
| Conductor radius | 1.7 mm |
| Normal distance between conductor and slab | 7.35 mm |

**TABLE 2**

Comparison Between Practical and Theoretical Results of "Ampere-Turn".

| Input power (Kw) | Current (A) | Frequency (KHz) | Percentage Error (% ) |
|---|---|---|---|
| 0.125 | 80.53 | 12 | 21.12 |
| 0.155 | 84.53 | 12.5 | 11.35 |
| 0.183 | 88.26 | 13 | 5.47 |

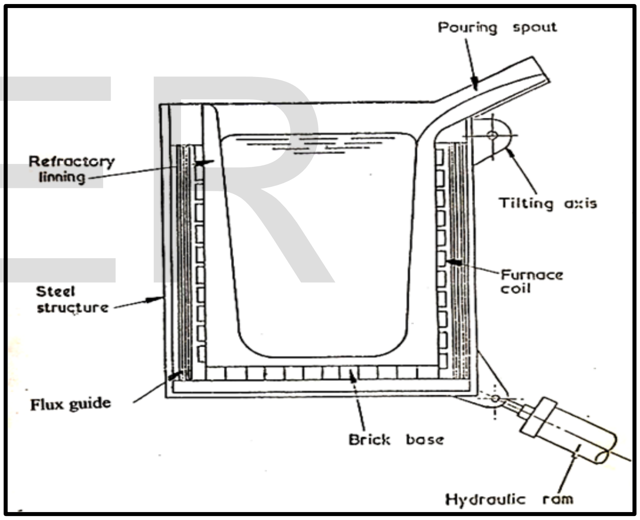


**Fig. (1)** Schematic illustration of a coreless furnace showing the pouring spout and tilting axis.

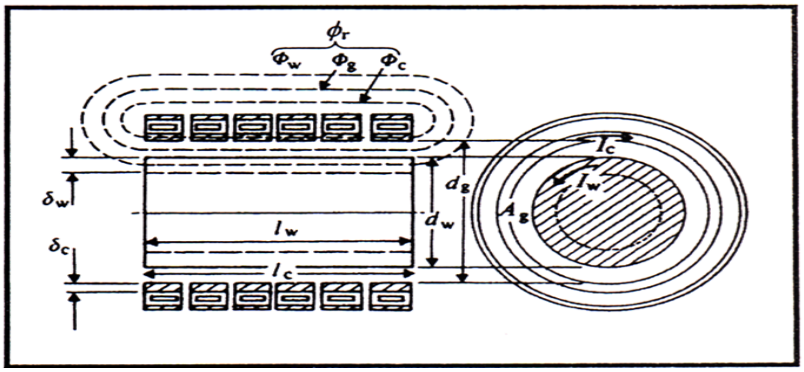


**Fig. (2)** The Flux paths in induction heating system.

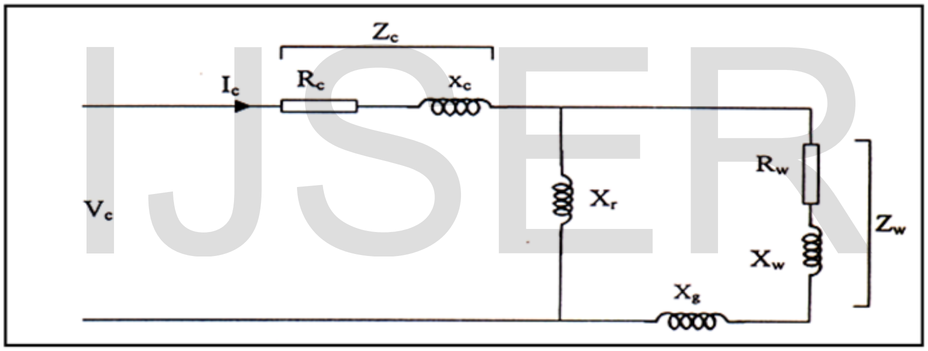


**Fig. (3)** The equivalent electrical circuit.

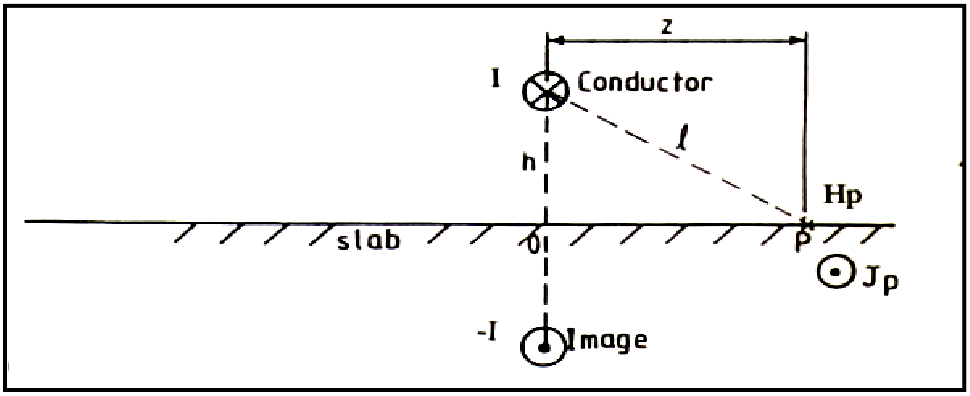
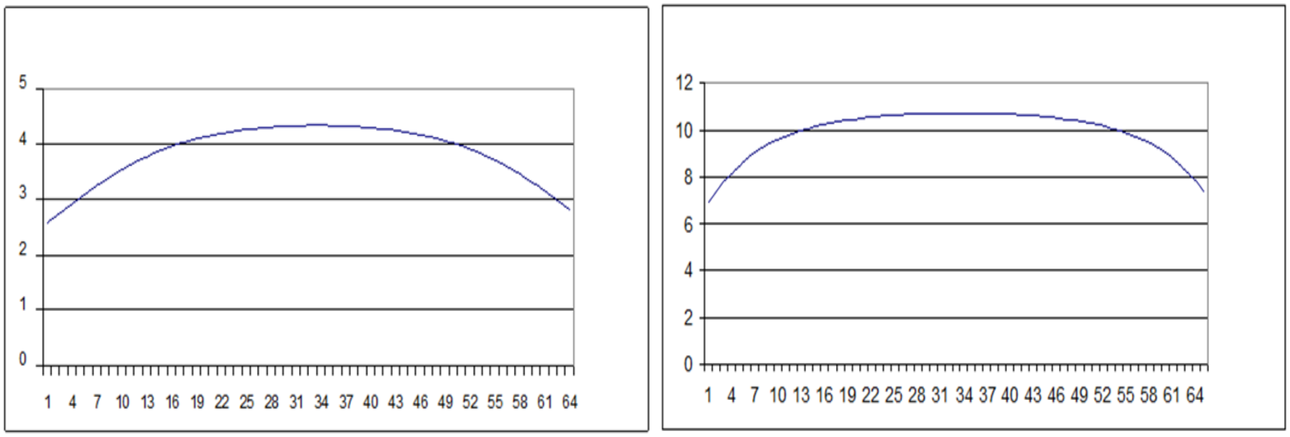
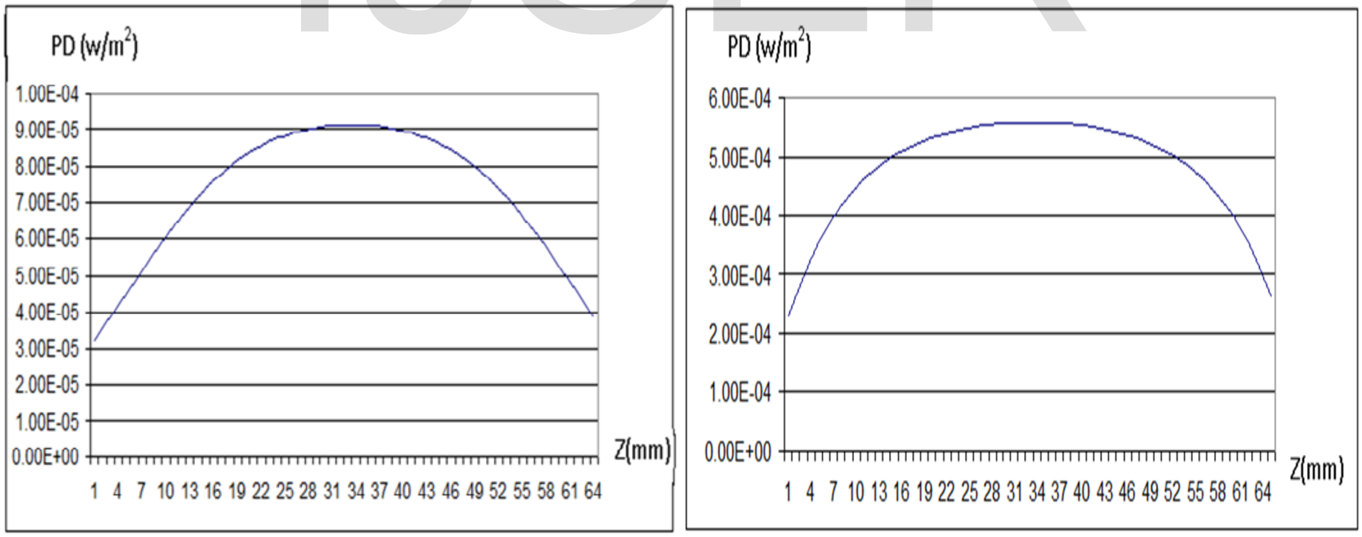


**Fig. (4)** Conductor near a semi- infinite slab.



**A** **B**

**Fig. (5)** Magnatic Field (H) (A/m) Against Furnace Length (Z)(mm)
I = 50A , n = 16 , F = 50 Hz
A - with α , β , Cos θ
B - without α , β , Cos θ



**A** **B**

**Fig. (6)** Power Density (PD) (W/m2) Against Furnace Length (Z)(mm)
I = 50A , n = 16 , F = 50 Hz
A - with α , β , Cos θ
B - without α , β , Cos θ

**TABLE 3**

Comparison of magnetic field (H) and power density (PD) between two cases:
With and without (α,β,Cosθ) , at I = 50 Amp. , and n = 16

| | | With α , β , Cos θ | | With out α , β , Cos θ | |
|---|---|---|---|---|---|
| Distance mm | Freq. Hz | H A/m | PD W/m2 | H A/m | PD W/m2 |
| 0 | 50 | 2.58576756131602 | 0.0000325247700443374 | 6.85415227945138 | 0.000228529761750665 |
| 0 | 3000 | 2.58576756131602 | 0.000274832332191818 | 6.85415227945137 | 0.00177018392274615 |
| 0 | 6000 | 2.58576756131602 | 0.000388671611564297 | 6.85415227945137 | 0.00250341811144241 |
| 0 | 12000 | 2.58576756131602 | 0.000503871570883779 | 6.85415227945137 | 0.0035403678454923 |
| 17 | 50 | 4.05195854860394 | 0.0000798666110771301 | 10.3534106242477 | 0.000521437362662646 |
| 17 | 3000 | 4.05195854860394 | 0.0006186441092395 | 10.3534106242477 | 0.00403903644336549 |
| 17 | 6000 | 4.05195854860394 | 0.000874894889568723 | 10.3534106242477 | 0.00571206011712667 |
| 17 | 12000 | 4.05195854860394 | 0.001237288218479 | 10.3534106242477 | 0.00807807288673099 |
| 33 | 50 | 4.34051667642463 | 0.0000916469734094605 | 10.7148864018856 | 0.000558483582719811 |
| 33 | 3000 | 4.34051667642463 | 0.000709894403490309 | 10.7148864018856 | 0.00432599523000816 |
| 33 | 6000 | 4.34051667642463 | 0.00100394229326875 | 10.7148864018856 | 0.00611788112503885 |
| 33 | 12000 | 4.34051667642463 | 0.00141978880698062 | 10.7148864018856 | 0.00865199046001631 |
| 64 | 50 | 2.70071318425421 | 0.0000354807015192753 | 7.34025794631178 | 0.000262094503135043 |
| 64 | 3000 | 2.70071318425421 | 0.000274832332191818 | 7.34025794631178 | 0.00203017529154906 |
| 64 | 6000 | 2.70071318425421 | 0.000388671611564297 | 7.34025794631178 | 0.00287110143130343 |
| 64 | 12000 | 2.70071318425421 | 0.000549664664383637 | 7.34025794631 | 0.00406035058309812 |

### TABLE 4

Comparison of magnetic field (H) and power density (PD) between two cases:
with and without (α , β , Cos θ ) , at I = 100 Amp. , and n = 16

| | | With<br>α , β , Cos θ | | With out<br>α , β , Cos θ | |
|---|---|---|---|---|---|
| Distance mm | Freq. Hz | H A/m | PD W/m2 | H A/m | PD W/m2 |
| 0 | 50 | 5.17153512263203 | 0.00013009908017735 | 13.7083045589027 | 0.000914119047002661 |
| 0 | 3000 | 5.17153512263203 | 0.00100774314176756 | 13.7083045589027 | 0.0070807356909846 |
| 0 | 6000 | 5.17153512263203 | 0.00142516401847615 | 13.7083045589027 | 0.0100136724457697 |
| 0 | 12000 | 5.17153512263203 | 0.00201548628353512 | 13.7083045589027 | 0.0141614713819692 |
| 17 | 50 | 8.0110847833735 | 0.000312189236031844 | 20.5877777096231 | 0.00206183643486423 |
| 17 | 3000 | 8.10391709720788 | 0.002474576436958 | 20.7068212484955 | 0.0159709163496657 |
| 17 | 6000 | 8.10391709720788 | 0.00349957955827489 | 20.7068212484955 | 0.0228482404685067 |
| 17 | 12000 | 8.10391709720788 | 0.004949152873916 | 20.7068212484955 | 0.032312291546924 |
| 33 | 50 | 8.68103335284926 | 0.000366587893637842 | 21.4297728037711 | 0.00223393433087924 |
| 33 | 3000 | 8.68103335284926 | 0.00283957761396124 | 21.4297728037711 | 0.0173039809200326 |
| 33 | 6000 | 8.68103335284926 | 0.00401576917307502 | 21.4297728037711 | 0.0244715245001554 |
| 33 | 12000 | 8.68103335284926 | 0.00567915522792247 | 21.4297728037711 | 0.0346079618400653 |
| 64 | 50 | 5.40142636850842 | 0.000141922806077101 | 14.6805158926236 | 0.00104837801254017 |
| 64 | 3000 | 5.40142636850842 | 0.00109932932876727 | 14.6805158926236 | 0.00812070116619624 |
| 64 | 6000 | 5.40142636850842 | 0.00155468644625719 | 14.6805158926236 | 0.0114844057252137 |
| 64 | 12000 | 5.40142636850842 | 0.00219865865753455 | 14.6805158926236 | 0.0162414023323925 |

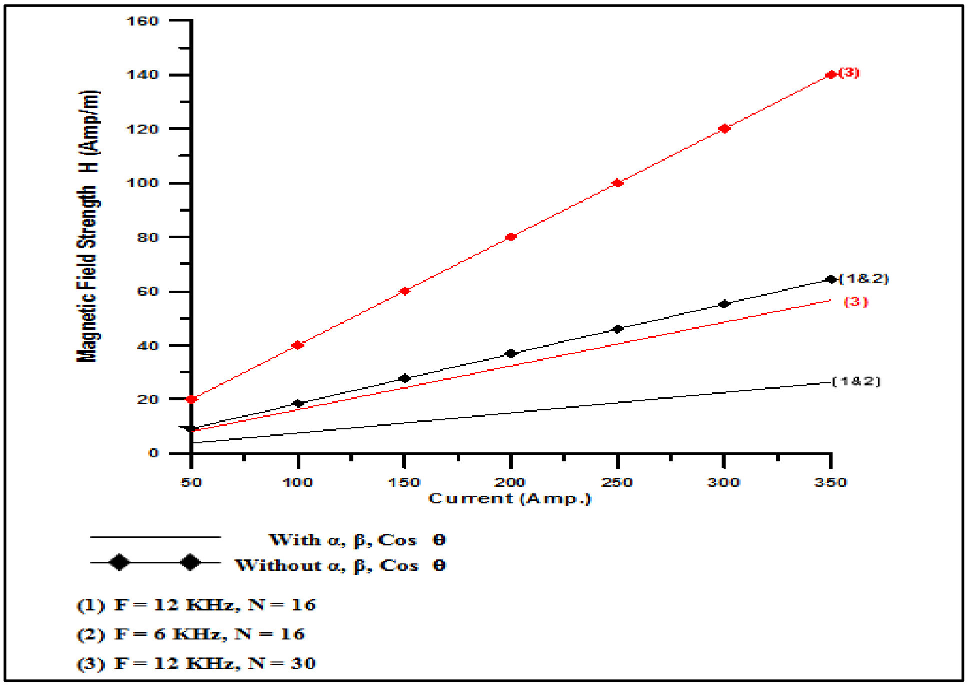


**Fig. (7)** Magnetic field strength (H) versus current (I) at the center.

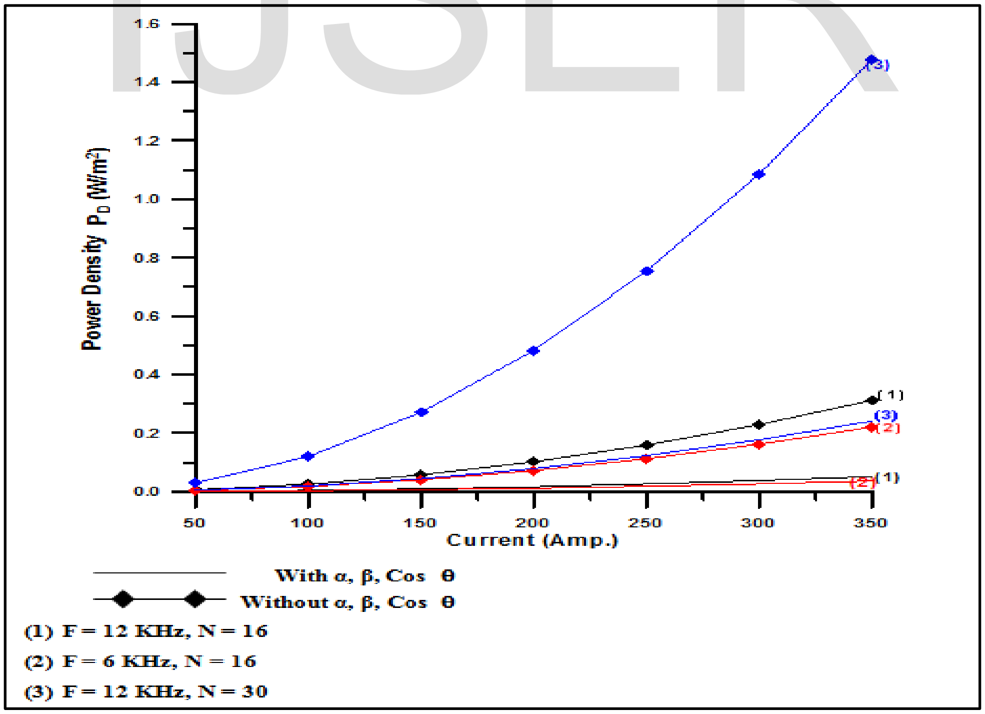


**Fig. (8)** Power density (PD) versus current (I), at the center

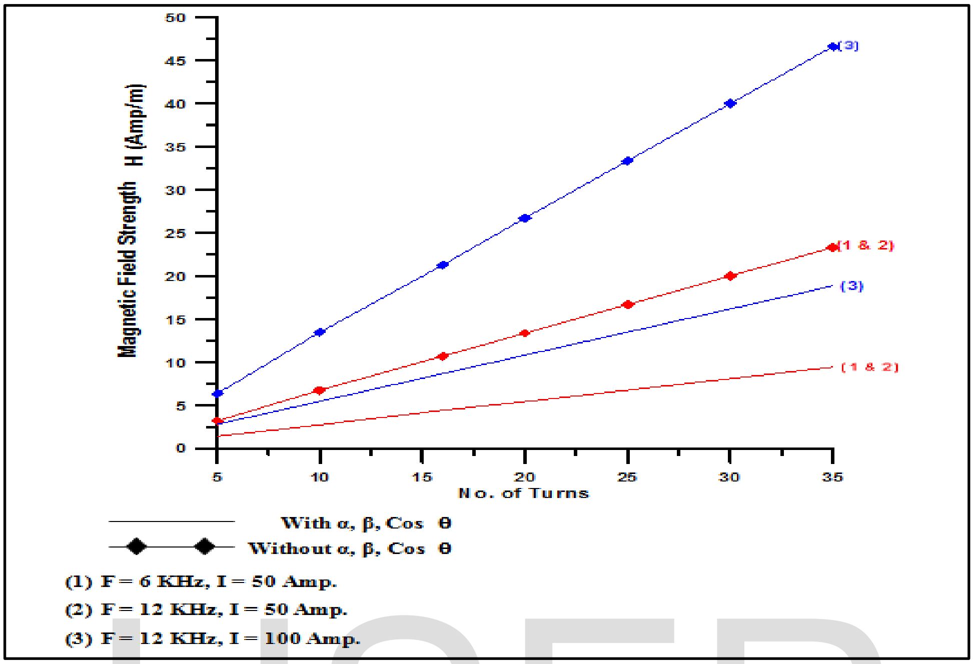


**Fig. (9)** Magnetic field strength (H) versus number of turns (N), at the center.

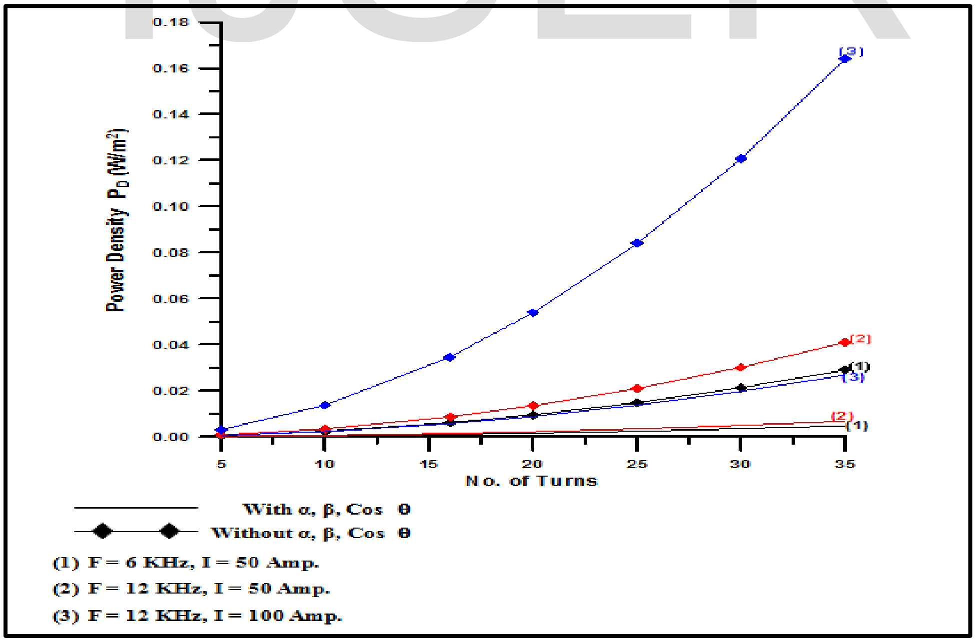


**Fig. (10)** Power density (PD) versus number of turns (N), at the center.

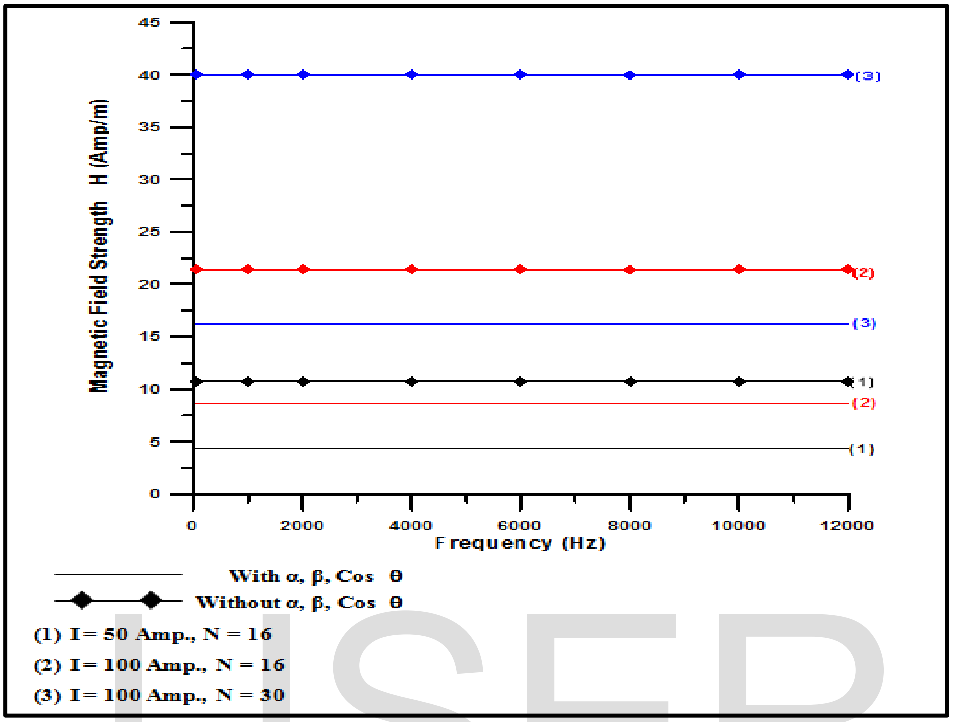


**Fig. (11)** Magnetic field strength (H) versus frequency (F), at the center.

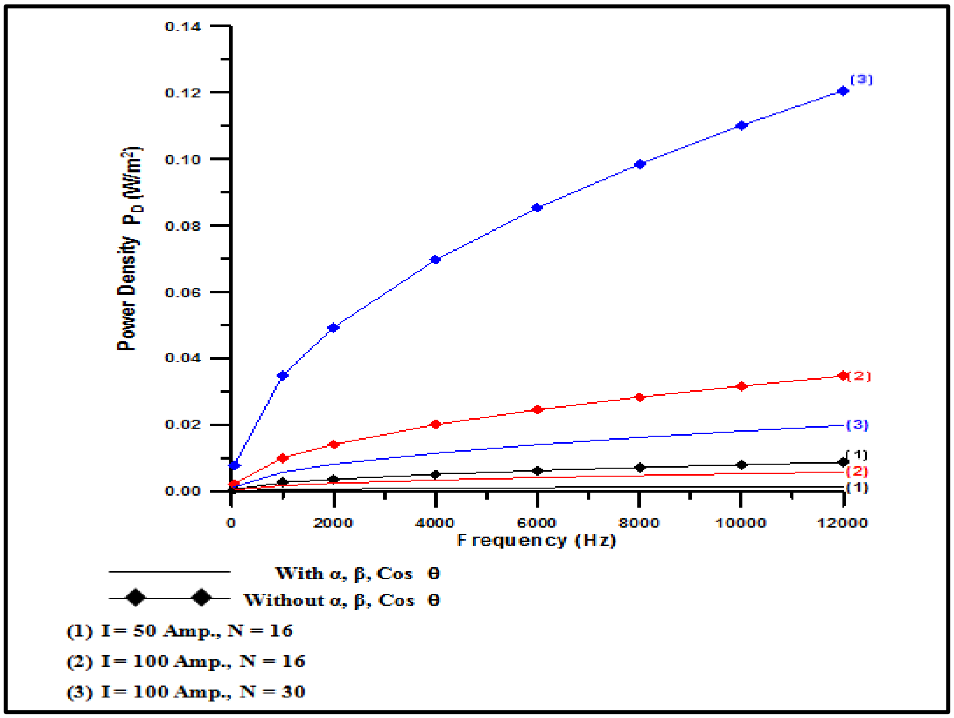


**Fig. (12)** Power density ($P_D$) versus frequency (F), at the center.